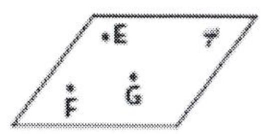
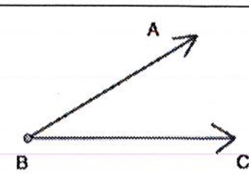
WLPCS
Geometry

Name: ANSWER KEY Date: ____________ Per.: ________

FINAL REVIEW – Basic Geometry Vocabulary

Euclid:

- the father of Geometry
- studied by Abraham Lincoln
- built an *axiomatic* system of Geometry
    - based on **axioms** – statements accepted as true
    - ex: A straight line segment can be drawn joining any two points.

	Description	Figure	Symbol
* **point**	describes a location; zero dimensions		P or Point P
* **line**	a collection of points along a straight path with no endpoints; one dimension (length)		$\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$
* **plane**	a flat surfaces that extends indefinitely; two dimensions (length and width)		Plane EFG or Plane T
ray	a collection of points along a straight path with one endpoint which extends indefinitely in one direction; one dimension (length)		$\overrightarrow{PQ}$
line segment	a collection of points along a straight path with two endpoints; one dimension (length) **measurable*		$\overline{XY}$ or $\overline{YX}$
angle	two rays that meet at a point (this point is the **vertex**) **measurable*		$\angle ABC$

* These are the UNDEFINED TERMS in Geometry... many vocabulary words we use depend on one or more of these 3 words.

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Directions: I HIGHLY recommend answering these on a separate sheet of paper.

segment, ray PQ, line PQ, ray QP

1. Describe what each of these symbols means: $\overline{PQ}$, $\overrightarrow{PQ}$, $\overleftrightarrow{PQ}$, $\overrightarrow{QP}$.

2. Sketch a line that contains point R between points S and T. Which of the following are true?

A. $\overrightarrow{SR}$ is the same as $\overrightarrow{ST}$. T

B. $\overrightarrow{SR}$ is the same as $\overleftrightarrow{RT}$. T

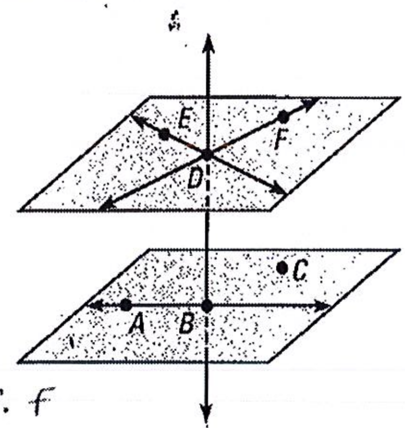
C. $\overrightarrow{RS}$ is the same as $\overrightarrow{TS}$. F

D. $\overrightarrow{RS}$ and $\overrightarrow{RT}$ are opposite rays. T

E. $\overline{ST}$ is the same as $\overline{TS}$. T

F. $\overrightarrow{ST}$ is the same as $\overrightarrow{TS}$. F

Decide whether the statement is *true* or *false*.



3. Points A, B, and C are collinear. F

4. Points A, B, and C are coplanar. T

5. Point F lies on $\overrightarrow{DE}$. F

6. $\overleftrightarrow{DE}$ lies on plane DEF. T

7. $\overleftrightarrow{BD}$ and $\overleftrightarrow{DE}$ intersect. T

8. $\overleftrightarrow{BD}$ is the intersection of plane ABC and plane DEF. F

Short Answer:

1. What are the undefined terms?

Point, line, plane

2. Why will two points ALWAYS be collinear? (1) Why will three points always be coplanar? (2)

(1) B/c there is always a unique line that will connect any two points

(2) B/c for any 3 points, there exists a plane that contains all 3 points

3. In what way(s) was Euclid influential?

- built/organized the system of Geometry based on a set of axioms (statements accepted as true w/o proof)
- Lincoln studied Euclid to build his skills in logic to use toward making a good, structured argument

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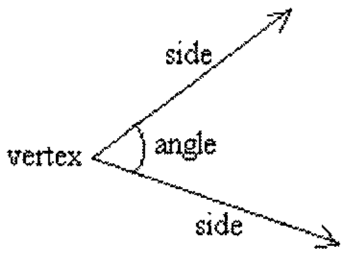
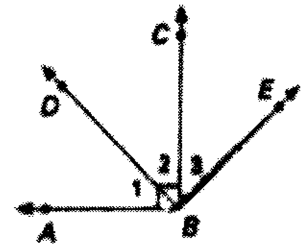
FINAL REVIEW – Angles

What is an angle?



*the angle is the space between the two sides… a portion of 360°

How do you name an angle?

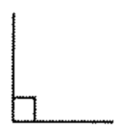


$\angle 1$ can also be named $\angle ABD$ or $\angle DBA$
$\angle 2$ can also be named $\angle DBC$ or $\angle CBD$

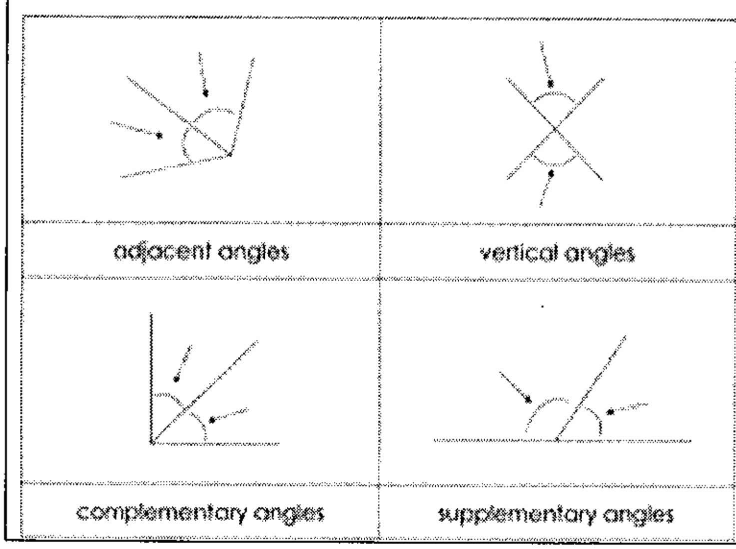
notice that the VERTEX is always the letter in the middle

How can you describe an angle?

Acute angle	*Right angle*	*Obtuse angle*	*Straight line*	*Reflex angle*
less than 90°	= 90°	between 90° and 180°	= 180°	greater than 180°



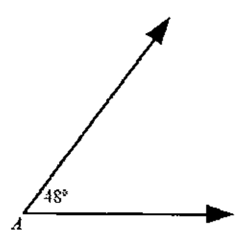
Angle relationships

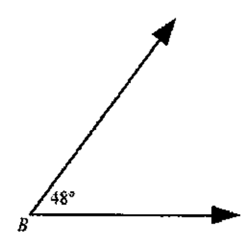


Congruent Angles

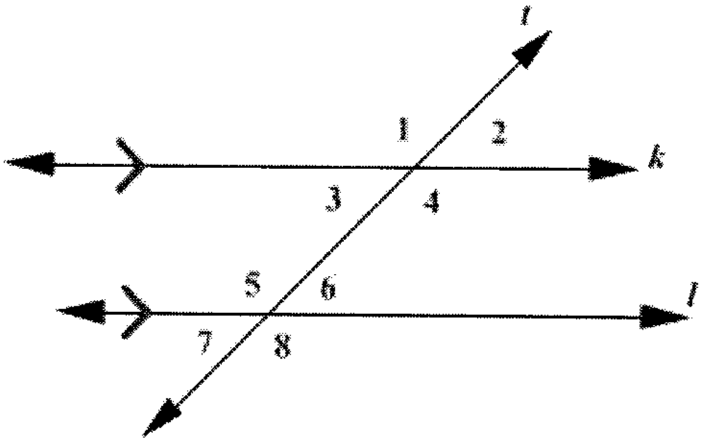
Congruent angles have the same angle measure.

$\angle A \cong \angle B$ because the measure of both angles is 48°.





Parallel Lines cut by a Transversal



In the diagram to the left, line *l* and line *k* are parallel. Line *t* is a **transversal** because it cuts through both lines.

Key Points:

1. Vertical angles are congruent.
Ex: $\angle 1 \cong \angle 4$ and $\angle 7 \cong \angle 6$

2. Alternate interior angles are congruent.
Ex: $\angle 3 \cong \angle 6$ and $\angle 5 \cong \angle 4$

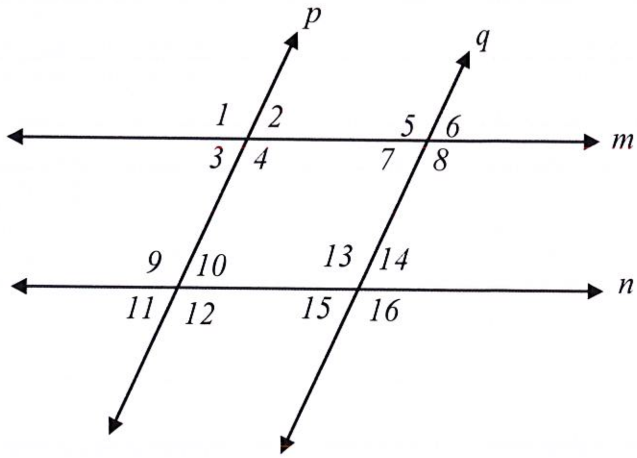
3. Corresponding angles are congruent.
Ex: $\angle 1 \cong \angle 5$ and $\angle 8 \cong \angle 4$

4. Same side interior angles are supplementary.
Ex: $m\angle 6 + m\angle 4 = 180°$

5. Same side exterior angles are supplementary.
Ex: $m\angle 8 + m\angle 2 = 180°$

6. Linear pairs are supplementary.
Ex: $m\angle 1 + m\angle 2 = 180°$

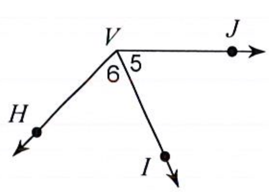
FINAL REVIEW – Angles: Problem Set



Using the diagram above and given that line *m* is parallel to line *n:*

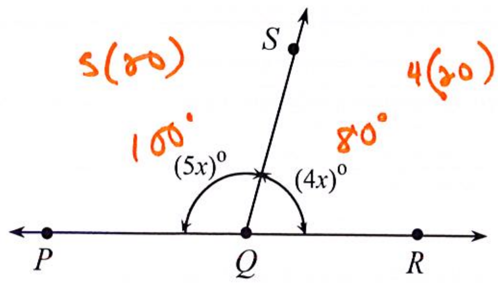
1. Name a pair of alternate interior angles.
   Many answers Ex: ∠8 + ∠13, ∠7 + ∠14, ∠4 + ∠9
2. Name a pair of corresponding angles.
   Many answers Ex: ∠2 + ∠10, ∠11 + ∠3
3. Name a pair of vertical angles.
   Many answers Ex: ∠1 + ∠4, ∠13 + ∠16, ∠15 + ∠14
4. If $m\angle 11 = 80^{\circ}$, what is the measure of $\angle 10$? How do you know?
   m∠10 = 80° b/c they are vertical angles, which are congruent to each other
5. If $m\angle 11 = 80^{\circ}$, what is the measure of $m\angle 12$? How do you know?
   m∠12 = 100° b/c linear pairs form supplementary angles
6. If If $m\angle 2 = 65^{\circ}$ and If $m\angle 5 = 115^{\circ}$, are lines *p* and *q* parallel? Explain why or why not.
   Yes! These are <u>same side</u> <u>interior</u> angles. If they are <u>supplementary</u>, the lines are parallel. (65°+115°=180°)

6. Using the diagram below, name three angles with the vertex V.



∠JVI, ∠HVI, ∠HVJ,
∠5, ∠6

In the figure on the right, points P, Q and R are collinear. What is the measure of $\angle RQS$?



A. 40° B. 20° C. 80°

D. 50° E. 100°

$9x = 180$
$x = 20$

8.

If $\angle A$ and $\angle B$ are complementary, $\angle B$ and $\angle C$ are supplementary, and $m\angle A = 64°$, then what is the measure of $\angle C$?

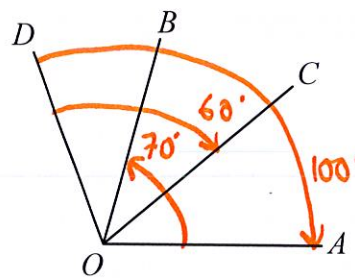
$m\angle A + m\angle B = 90°$
$64 + m\angle B = 90°$
$m\angle B = 26°$

$m\angle B + m\angle C = 180°$
$26° + m\angle C = 180$
$m\angle C = 154°$

A. 64° B. 180° C. 26° D. 90° E. 154°

9.

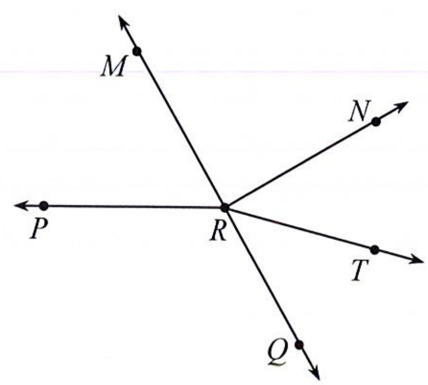
In this figure, $m\angle AOB = 70°$, $m\angle COD = 60°$, and $m\angle AOD = 100°$. What is $m\angle COB$?



A. 10° B. 65° C. 35°

D. 60° E. 30°

Problems 14-16: Refer to the figure on the right, in which M, R and Q are collinear and $m\angle MRN = 90°$:



C 14. Which of the following is a *straight angle*?

A. $\angle MRN$ B. $\angle PMR$ C. $\angle MRQ$

D. $\angle PRN$ E. $\angle NTR$

B 15. Which of the following is an *obtuse angle*?

A. $\angle MRQ$ B. $\angle PRN$ C. $\angle NTR$ D. $\angle MRN$ E. $\angle PMR$

A 16. Which of the following angles is *adjacent* to $\angle NRT$?

A. $\angle QRT$ B. $\angle MRT$ C. $\angle PRM$ D. $\angle PRN$ E. $\angle PMR$

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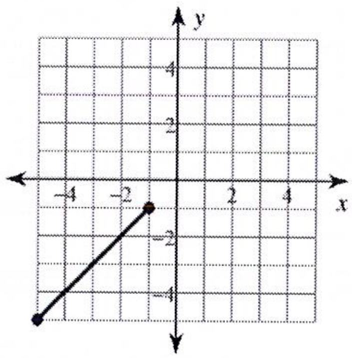
FINAL REVIEW – Midpoint, Distance, and Partitioning a Line Segment

The Formula:	Explained:
$midpoint = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	The midpoint is the average of the x-coordinates and the average of the y-coordinates.

Find the midpoint of each line segment.



Coordinates of the endpoints:

(-5, -5)
(-1, -1)

$$\left(\frac{-5+-1}{2}, \frac{-5+-1}{2}\right)$$

(-3, -3)

11) (2, 4), (1, −3)

$$\left(\frac{2+1}{2}, \frac{4+-3}{2}\right)$$

(1.5, 0.5)

12) (−4, 4), (−2, 2)

$$\left(\frac{-4+-2}{2}, \frac{4+2}{2}\right)$$

(-3, 3)

The Formula:	Explained:
$distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Derived from the Pythagorean Theorem

Find the **length** of the line segment with the following endpoints OR find the **distance** between the following two points: (3, 7) and (2, 9).

$x_1\ y_1 \quad x_2\ y_2$

$$\sqrt{(2-3)^2 + (9-7)^2}$$

$$\sqrt{(-1)^2 + (2)^2}$$

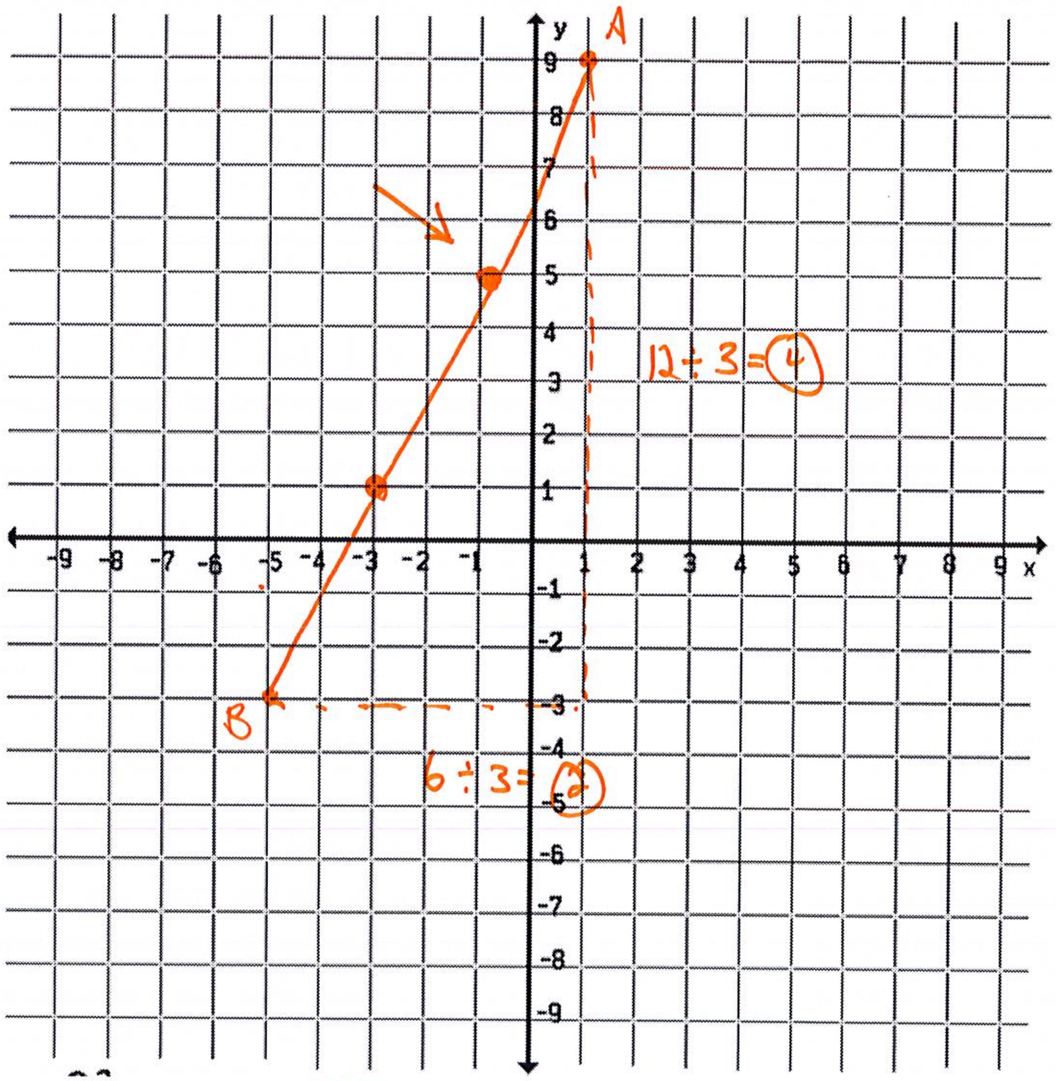
$$\sqrt{1+4}$$

$$\sqrt{5}$$

Partitioning a Line Segment

3 parts

Find the coordinates of Point P on a directed line segment AB in a ratio of 1:2. A(1, 9) B(-5, -3)



(-1, 5)

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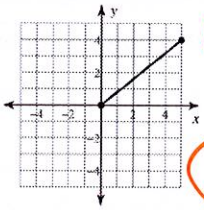
Directions: Find the midpoint of each segment.

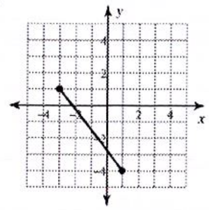
5)



(0,0) and (5,4)

(2.5,2)

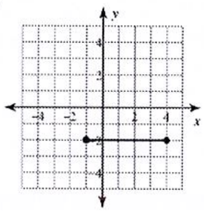
6)



(-3,1) and (1,-4)

(-1,-1.5)

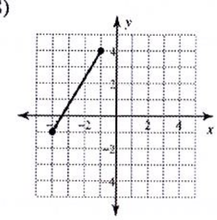
7)



(-1,-2)and (4,-2)

(1.5,-2)

8)



(-4,-1) and (-1,4)

(-2.5,1.5)

Find the midpoint of the line segment with the given endpoints.

9) $(-4, 4)$, $(5, -1)$

(0.5,1.5)

10) $(-1, -6)$, $(-6, 5)$

(-3.5,-0.5)

11) $(2, 4)$, $(1, -3)$

(1.5, 0.5)

12) $(-4, 4)$, $(-2, 2)$

(-3, 3)

Directions: Find the length of each line segment.

7.) $(3, 7)$ and $(2, 9)$

$\sqrt{5}$

8.) $(-6, 11)$ and $(4, -1)$

$\sqrt{244}$

9.) $(0, -8)$ and $(-19, 2)$

$\sqrt{461}$

10.) $(-4, 2)$ and $(-4, 11)$

$\sqrt{81} = 9$

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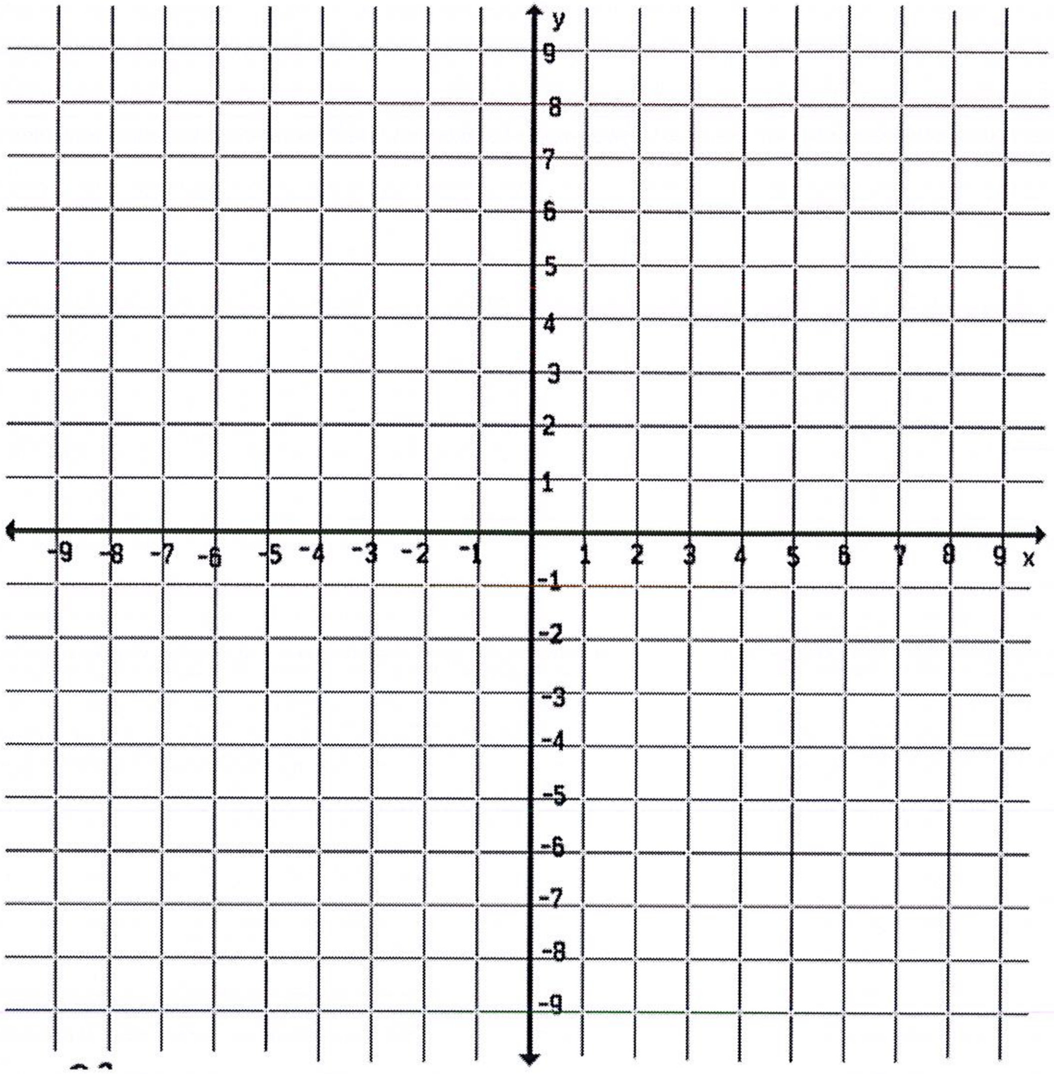

Directions: Given two endpoints, partition the line segment in the ratio indicated. Write your answer as a coordinate point.

1. Find the coordinates of point P on directed line segment AB that partition AB in the ratio 1:1.
A (-3, 4) B (7, 6)

(2,5)

2. Find the coordinates of point P on directed line segment BA that partition BA in the ratio 2:3
A (-9, 3) B (1, 8)

(-3,6)

3. Find the coordinates of point P on directed line segment AB that partition AB in the ratio 1:3.
A (8, -5) B (4, 7) 1:3

(7,-2)

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FINAL REVIEW – Transformations

REFLECTION – 1. a FLIP across an axis of symmetry
2. CONGRUENCE TRANSFORMATION

Reflection in the **x-axis:**	When you reflect a point across the x-axis, the x-coordinate remains the same, but the y-coordinate is transformed into its opposite. $P(x,y) \rightarrow P'(x,-y)$ or $r_{x\text{-}axis}(x,y) = (x,-y)$
Reflection in the **y-axis:**	When you reflect a point across the y-axis, the y-coordinate remains the same, but the x-coordinate is transformed into its opposite. $P(x,y) \rightarrow P'(-x,y)$ or $r_{y\text{-}axis}(x,y) = (-x,y)$
Reflection in $y = x$:	When you reflect a point across the line $y = x$, the x-coordinate and the y-coordinate change places. $P(x,y) \rightarrow P'(y,x)$ or $r_{y=x}(x,y) = (y,x)$
Reflection in $y = -x$:	When you reflect a point across the line $y = -x$, the x-coordinate and the y-coordinate change places and are negated (the signs are changed). $P(x,y) \rightarrow P'(-y,-x)$ or $r_{y=-x}(x,y) = (-y,-x)$

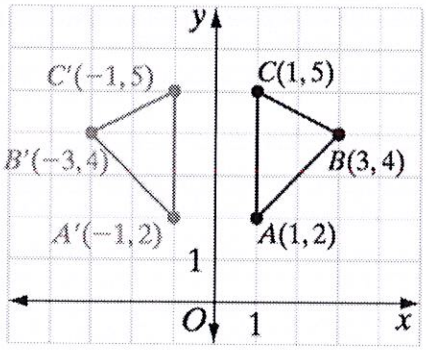
ROTATION: 1. A rotation turns a figure through an angle about a fixed point called the center.
2. CONGRUENCE TRANSFORMATION

Rotation of **90°:**	$R_{90^\circ}(x,y) = (-y,x)$
Rotation of **180°:**	$R_{180^\circ}(x,y) = (-x,-y)$ (same as point reflection in origin)
Rotation of **270°:**	$R_{270^\circ}(x,y) = (y,-x)$

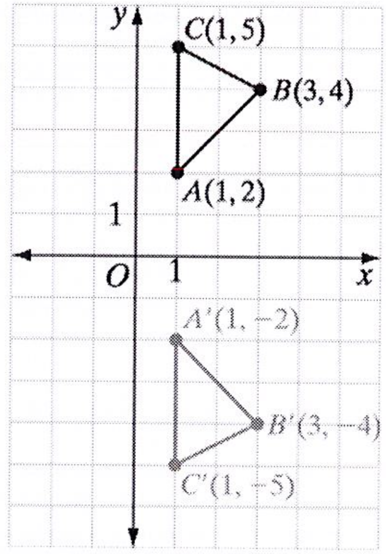
TRANSLATION: 1. A translation "slides" an object a fixed distance in a given direction.
2. CONGRUENCE TRANSFORMATION

Translation of **h, k:**	$T_{h,k}(x,y) = (x+h, y+k)$

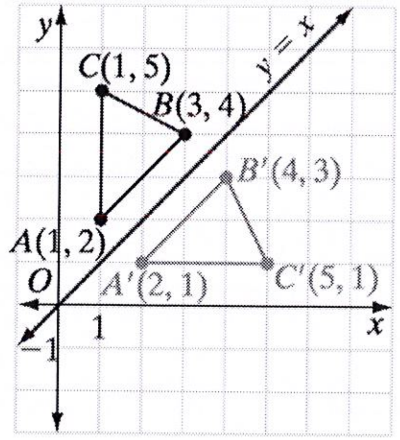
Reflection in the y-axis:



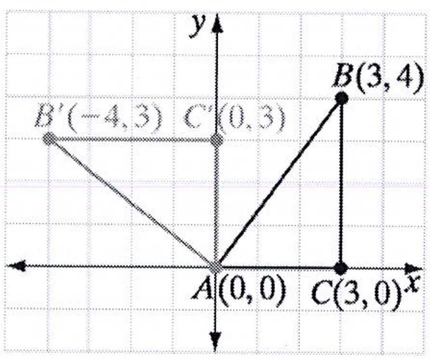
Reflection in the *x*-axis:



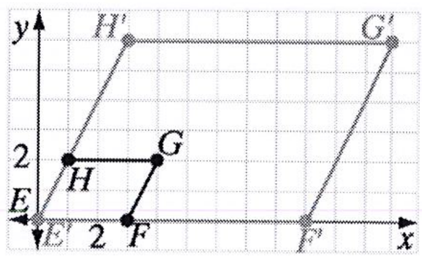
Reflection in y = x:



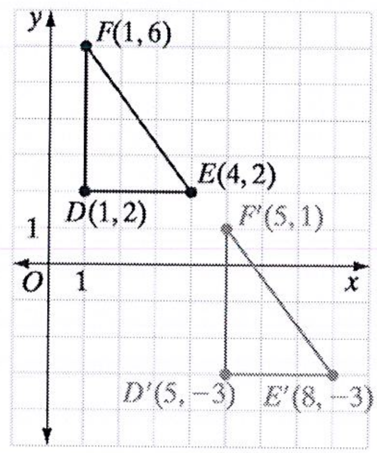
Rotation:
Counter-Clockwise 90-degrees



Dilation (center is (0, 0):

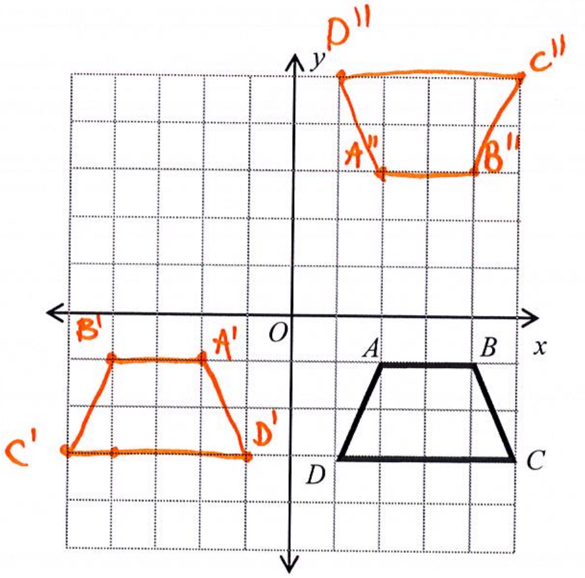


Translation:



- Rotations, reflections, and translations are congruence transformations b/c the pre-image and image remain congruent to each other.
- The pre-image can be mapped onto the image in a congruence transformation.

FINAL REVIEW – Transformations



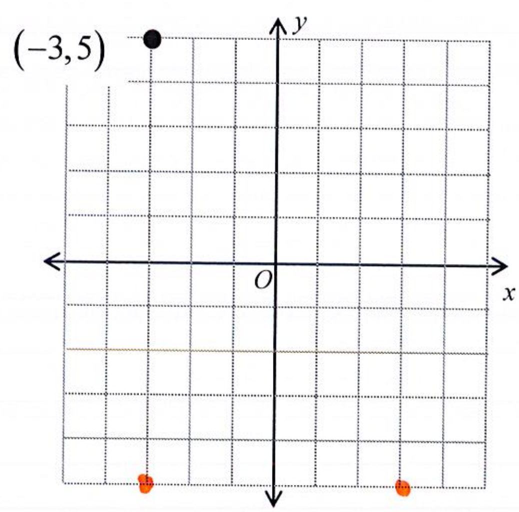
A. Reflect the figure across the y-axis. Name the image *A'B'C'D'*.

B. Does the image have the same side lengths and angle measurements? Justify your answer.

Yes! Reflection is a congruence transformation (the image and pre-image are congruent)

C. If the figure was reflected across the *x*-axis, then translated two units upward, would the result be the same as the transformation in part a? If not, show were the image would be on the coordinate plane and label it *A''B''C''D''*.

#2



If the pre-image point is (-3,5), write the coordinates of the image point after it undergoes the following transformations:

a. Reflected across the *x*-axis (-3,-5)

b. Reflected across the *y*-axis (3,-5)

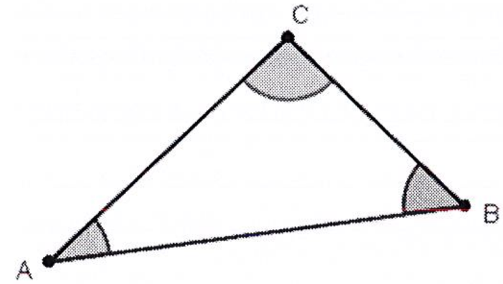
c. Translated seven units to the right and two units downward [(*x*+7, *y*-2)] (10,-7)

d. Rotated counter-clockwise 90 degrees about the origin. (7,10)

e. Rotated 180 degrees about the origin. (-7,-10)

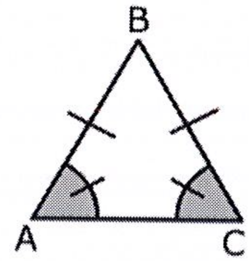
FINAL REVIEW - Triangles

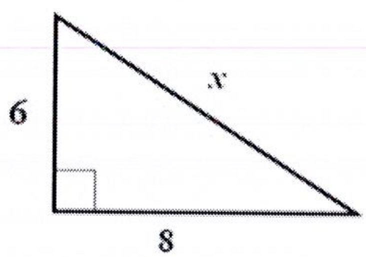
- The sum of the interior angles of ANY triangle is 180°.
- Every triangle has three sides and three angles.

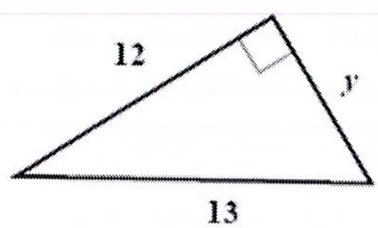


$$m\angle A + m\angle B + m\angle C = 180$$

- Classification:
  - Equilateral Triangle: all side lengths are equal
    - each interior angle is 60°
  - Isosceles Triangle: two side lengths are equal
    - base angles are congruent (see image to the right →)
  - Scalene Triangle: no equal side lengths
  - Right Triangle: one angle is a right angle, the other two angles are acute (smaller than 90°)
    - PYTHAGOREAN THEOREM! Remember: If you have a right triangle, you can use Pythagorean Theorem to find the missing side. (Examples below)







$$6^2 + 8^2 = x^2$$
$$36 + 64 = x^2$$
$$100 = x^2$$
$$\sqrt{100} = \sqrt{x^2}$$
$$\boxed{x = 10}$$

$$12^2 + y^2 = 13^2$$
$$144 + y^2 = 169$$
$$y^2 = 25$$
$$\sqrt{y^2} = \sqrt{25}$$
$$\boxed{y = 5}$$

**** NO PROBLEM SET****

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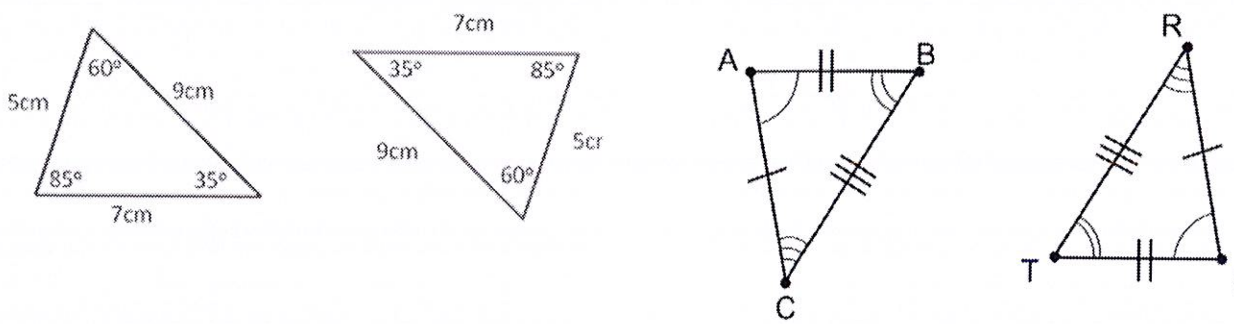
FINAL REVIEW - Triangle Congruence

- Two figures are congruent if all **corresponding lengths** are the same, and if all **corresponding angles** have the **same measure**.

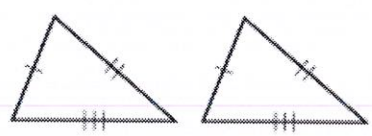
- same shape, same size

- Both pairs of triangles below are congruent. You can tell the first pair is congruent because corresponding sides and angles are the same measure. You can tell the second pair is congruent because of the congruence markings.



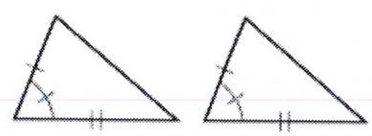
- Proving Triangles Congruent

Side-Side-Side (SSS)



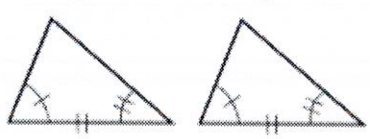
Three pairs of congruent sides

Side-Angle-Side (SAS)



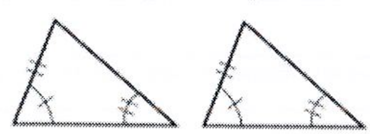
Two pairs of congruent sides and one pair of congruent angles (angles between the pairs of sides)

Angle-Side-Angle (ASA)



Two pairs of congruent angles and one pair of congruent sides (sides between the pairs of angles)

Side-Angle-Angle (SAA)



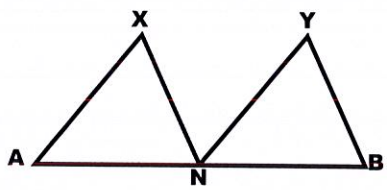
Two pairs of congruent angles and one pair of congruent sides (sides not between the pairs of angles)

CAREFUL! AAA (Angle Angle Angle) and **SSA** (Side Side Angle) do **<u>NOT</u>** prove two triangles congruent! DO NOT USE THEM IN A PROOF!

WLPCS
Geometry

- Important Concepts for Proofs

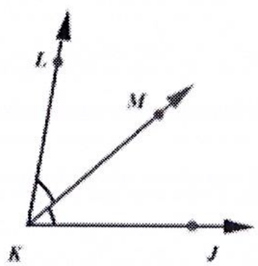
  - Midpoint: the middle point of a line segment; It is equidistant from both endpoints; it bisects the segment.



In the diagram to the left:

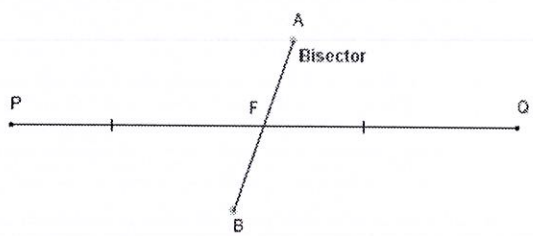
***N* is the midpoint of $\overline{AB}$ so...**
$\overline{AN} \cong \overline{BN}$

  - Bisector: a line that cuts an angle or line segment into two equal parts



In the diagram to the left:

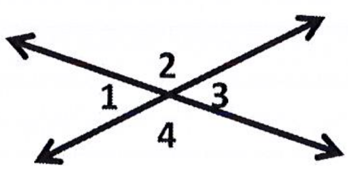
$\overrightarrow{KM}$ **bisects** $\angle LKJ$ **so...** $\angle LKM \cong \angle JKM$



In the diagram to the left:

$\overline{AB}$ **bisects** $\overline{PQ}$ **so...** $\overline{PF} \cong \overline{QF}$

  - Vertical angles



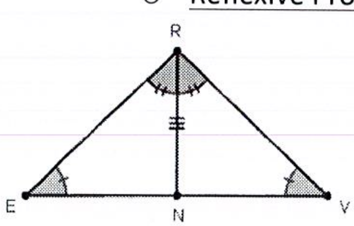
In the diagram to the left:

Angles 1 and 3 are vertical angles so they are CONGRUENT.

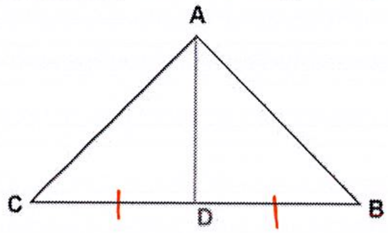
Angles 2 and 4 are vertical angles so they are CONGRUENT.

  - Corresponding Parts of Congruent Triangles are Congruent (CPCTC): If two triangles are congruent, then the corresponding parts of those triangles are congruent!

  - Reflexive Property – in the diagram below $\overline{RN} \cong \overline{RN}$ by the reflexive property



Directions: Use the diagram to answer the questions below.



$\overline{AD}$ bisects $\overline{CB}$; therefore, $\overline{CD} \cong \overline{BD}$. You are asked to prove $\triangle ADC \cong \triangle ADB$.

a. To prove congruence by SSS, what two additional congruence statements are needed?

1. $\overline{AC} \cong \overline{AB}$ 2. $(\overline{AD} \cong \overline{AD})$

b. To prove congruence by SAS, what two additional congruence statements are needed?

1. $\angle ADC \cong \angle ADB$ 2. $(\overline{AD} \cong \overline{AD})$

c. To prove congruence by ASA, what two additional congruence statements are needed?

1. $\angle C \cong \angle B$ 2. $\angle ADC \cong \angle ADB$

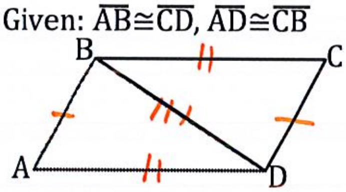
d. To prove congruence by AAS, what two additional congruence statements are needed?

1. $\angle C \cong \angle B$ 2. $\angle CAD \cong \angle BAD$

Directions: Fill in the blanks.

1.

Given: $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$

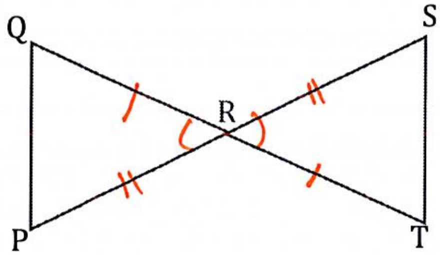


Prove: $\triangle ABD \cong \triangle BCD$

Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$	1. Given
2. $\overline{AD} \cong \overline{BC}$	2. Given
3. $\overline{BD} \cong \overline{BD}$	3. Reflexive Prop.
4. $\triangle ABD \cong \triangle BCD$	4. ~~SA~~ SSS

WLPCS
Geometry

2.
Given: $\overline{QR} \cong \overline{TR}$
$\overline{PR} \cong \overline{SR}$



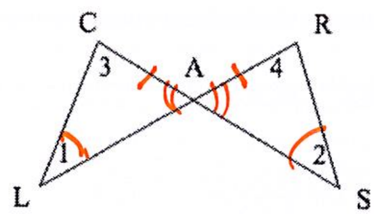
Prove: $\triangle QRP \cong \triangle SRT$

Statements	Reasons
1. $\overline{QR} \cong \overline{TR}$	1. Given
2. $\overline{PR} \cong \overline{SR}$	2. Given
3. $\angle SRT \cong \angle PRQ$	3. vertical angles are congruent
4. $\triangle QRP \cong \triangle SRT$	4. SAS

3.

Given: $\overline{AC} \cong \overline{AR}$ and $\angle 1 \cong \angle 2$

Prove: $\angle 3 \cong \angle 4$

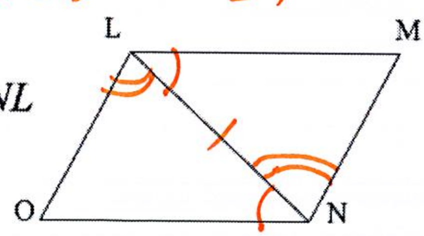


Proof:

1. $\overline{AC} \cong \overline{AR}$	1. Given
2. $\angle 1 \cong \angle 2$	2. Given
3. $\angle CAL \cong \angle RAS$	3. vertical angles are congruent
4. $\triangle LCA \cong \triangle SRA$	4. AAS
5. $\angle 3 \cong \angle 4$	5. CPCTC (corresponding parts of congruent triangles are $\cong$)

4.

Given: $\angle NLM \cong \angle LNO$ and $\angle OLN \cong \angle MNL$
Prove: $\angle M \cong \angle O$



Proof:

1. $\angle NLM \cong \angle LNO$	1. Given
2. $\angle OLN \cong \angle MNL$	2. Given
3. $\overline{LN} \cong \overline{LN}$	3. Reflexive Property of $\cong$
4. $\triangle LMN \cong \triangle NLM$	4. ASA
5. $\angle M \cong \angle O$	5. CPCTC